REPRESENTATION OF FUNCTIONS OF A COMPLEX VARIABLE AND RELATED INTEGRAL EQUATIONS

BY

WILLIAM ROLEN ORTON, JR.

A.B., University of Arkansas, 1947
A.M., University of Illinois, 1948

AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS
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REPRESENTATION OF FUNCTIONS OF A COMPLEX VARIABLE AND RELATED INTEGRAL EQUATIONS

In the works of V. Fedoroff[1] and W. Trjitzinsky[2] appear representations of the form

$$\text{(I)} \qquad f(z) = \int_D [u(t)/(t-z)]ds + A(z) \qquad z \text{ in } D.$$

Here $f(z)$ is a complex valued function of a suitable class; D is a bounded, connected region in the complex plane; ds is the differential element of area; $t = t' + it''$ is the variable point with respect to which integration is performed; $A(z)$ is an analytic function; and $u(z)$ is a kind of derivative of $f(z)$. An equation of the form (I) gives rise to a problem in integral equations as follows: suppose that $f(z)$ is a given function and $u(t)$ an unknown function in (I) where $A(z)$ is not given but is to be analytic; the problem is to fiind a $u(t)$ which satisfies the equation. It is clear that any *representation* of the form (I) for a class of functions $f(z)$ enables one to solve any *integral equation* of the form (I) involving a function $f(z)$ of that particular class.

The present work generalizes a representation theorem of V. Fedoroff by extending the class of functions representable by (I). This representation enables one to solve the corresponding integral equations. The paper then treats integral equations of the following type: let $f(z)$ and $k(z,t)$ be given functions and let $u(t)$ be an unknown function in

$$\text{(II)} \qquad f(z) = \int_D [k(z,t)\, u(t) / (t-z)]ds + A(z) \qquad z \text{ in } D$$

where $A(z)$ is not given but is to be analytic. Note that (II) differs from (I) in that a kernal $k(z,t)$ has been introduced. The main part of this work is an investigation of the solutions $u(t)$ of (II). Methods of W. Trjitzinsky[3] are used in placing conditions upon $f(z)$ and $k(z,t)$ which make (II) equivalent to a regular Fredholm integral equation of the second kind, quivalent in the sense that any solution of one is either a solution of the other or enables one to find a solution of the

other. The theory of the Fredholm equation being known, one can thus obtain information concerning equation (II). Several classes of functions f(z) are defined which yield solutions of (II) if the corresponding Fredholm equations have solutions. Relations are found between representations of the form (I) and ordinary derivatives of set functions defined by intergrals; these relations are used to obtain solutions of integral equations of the form (II). Representation theorems and equivalence theorems are also obtained for the case where the function A(z) in (I) and (II) is required only to be analytic almost everywhere in D. The investigation closes with a theorem which gives a series solution of (II) obtained by means of an iteration process.

(1) FEDOROFF, V., *O metode integral 'nykh srednikh i analogicnykh metodakh v teorii funkcij kompleksnogo.* Matematicheskii Sornik, Tom 2 (1937), pp.521-541.

(2) TRJITZINSKY, W., *Problems of Representation and Uniqueness for Functions of a Complex Variable.* Acta Mathematica, Vol. 78 (1946), pp.97-192.

(3) TRJITZINSKY, W., *Integral Equations in Problems of Representation of Functions of a Complex Variable.* Duke Mathematical Journal, Vol. 12 (1945), pp.419-440.

VITA

William Rolen Orton, Jr. was born on May 4, 1922, in Fulton, Arkansas. He attended Hope High School, Hope, Arkansas, through the junior year; the following year he received a $1,000 scholarship to attend Gulf Coast Military Academy (high school), Gulfport, Mississippi, from which he graduated in 1940. He attended the University of Arkansas for three years before being called to active duty in the Army of the United States in 1943. While serving as a platoon leader with the Fourth Infantry Division in Central Europe he was awarded the Bronze Star and Purple Heart medals. Upon his release from the army in February 1946, he again entered the University of Arkansas and was graduated the following year with the degree of Bachelor of Arts with high honors. During his senior year he was elected to membership in Phi Beta Kappa; for one semester he held a part time teaching assistantship. He entered the Graduate College of the University of Illinois in February 1947 and received the Master of Arts degree form that institution the following year. He gave instruction in mathematics at the University of Illinois as a part time assistant from February 1948 until June 1951.

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